RIGHT REVERSE DERIVATIONS ON PRIME RINGS

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ABSTRACT

In this paper some results concerning to right reverse derivations on prime rings with char $\neq 2$ are presented.

If $R$ be a prime ring with a non zero right reverse derivation $d$ and $U$ be the left ideal of $R$ then $R$ is commutative.

KEYWORDS: Prime Ring, Derivation, Reverse Derivation

INTRODUCTION

Bresar and Vukman [1] have introduced the notion of a reverse derivation. The reverse derivations on semi prime rings have been studied by Samman and Alyamani [2].

PRELIMINARIES

Throughout, $R$ will represent a prime ring with char $\neq 2$. We write $[x, y]$ for $xy - yx$. Recall that a ring $R$ is called prime if $aRb = 0$ implies $a = 0$ or $b = 0$. An additive mapping $d$ from $R$ into itself is called a derivation if $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$ and is called a reverse derivation if $d(xy) = d(y)x + yd(x)$ for all $x, y \in R$.

MAIN RESULTS

Theorem 1

Let $R$ be a prime ring with char $\neq 2$, $U$ a non-zero left ideal of $R$ and $d$ be a right reverse derivation of $R$. If $U$ is non-commutative such that $[x, d(x)] = 0$ for all $x \in U$, then $d = 0$.

Proof

By linearizing the equation $[d(x), x] = 0$ which gives

$[y, x]d(x) = 0$, for all $x, y \in U$ (1)

We replace $y$ by $zy$ in equ.(1) and using (1), we get,

$[zy, x]d(x) = 0$

$[z, x]y d(x) = 0$, for all $x, y, z \in U$ (2)

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By writing $y$ by $yr$, $r \in R$ in equation (2), we obtain,
\[ [z, x]yr \ d(x) = 0, \text{ for all } x, y, z \in U \text{ and } r \in R. \]

If we interchange $r$ and $y$, then we get,
\[ [z, x]ry \ d(x) = 0, \text{ for all } x, y, z \in U \text{ and } r \in R. \]

By primeness property, either $[z, x] = 0$ (or) $d(x) = 0$.

Since $U$ is non-commutative, then $d = 0$.

**Theorem 2**

Let $R$ be a prime ring with $\text{char} \neq 2$, $U$ a left ideal of $R$ and $d$ be a non-zero right reverse derivation of $R$.

If $[d(y), d(x)] = [y, x]$ for all $x, y \in U$, then $[x, d(x)] = 0$ and hence $R$ is commutative.

**Proof**

Given that $[d(y), d(x)] = [y, x]$, for all $x, y \in U$

By taking $yx$ instead of $y$ in the hypothesis, then we get,
\[ [yx, x] = [d(yx), d(x)] \]
\[ y[x, x] + [y, x]x = [(d(x)y + d(y)x), d(x)] \]
\[ [y, x]x = (d(x)y + d(y)x)d(x) – (d(x)y)d(x)y + d(y)x \]
\[ [y, x]x = d(x)yd(x) + d(y)x d(x) – d(x)d(y)x \]

Adding and subtracting $d(y)d(x)x$
\[ [y, x]x = d(x)yd(x) + d(y)x d(x) – d(x)d(y)x + d(y)d(x)x – d(y)d(x)x \]
\[ [y, x]x = d(x)yd(x) – d(x)d(y)x + d(y)x d(x) – d(y)d(x)x + d(y)d(x)x – d(y)d(x)x \]
\[ [y, x]x = d(x)[yd(x) – d(x)y] + d(y)x d(x) – d(x)x + [d(y)d(x) – d(x)d(y)]x \]
\[ [y, x]x = d(x)[y, d(x)] + d(y)[x, d(x)] + [d(y),d(x)]x \]
\[ [y, x]x = d(x)[y, d(x)] + d(y)[x, d(x)] + [y, x]x \]
\[ [y, x]x - [y, x]x = d(x)[y, d(x)] + d(y)[x, d(x)] \]
\[ d(x)[y, d(x)] + d(y)[x, d(x)] = 0, \text{ for all } x, y \in U \] (3)

We replace $y$ by $cy = yc$, where $c \in Z$ and using equation (3), we get,
\[ d(x)[cy, d(x)] + d(cy)[x, d(x)] = 0 \]
\[ d(x)(c[y, d(x)] + [c, d(x)]y) + (d(y)c + d(c)y)[x, d(x)] = 0 \]
\[ d(x)c[y, d(x)] + d(x)[c, d(x)]y + d(y)c[x, d(x)] + d(c)y[x, d(x)] = 0 \]
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\[ \Rightarrow c \ d(x)[y, d(x)] + d(x)[c, d(x)]y + c \ d(y)[x, d(x)] + d(c)y[x, d(x)] = 0 \]

\[ \Rightarrow -c \ d(y)[x, d(x)] + d(x)[c, d(x)]y + c \ d(y)[x, d(x)] + d(c)y[x, d(x)] = 0 \]

\[ \Rightarrow d(x)[c, d(x)]y + d(c)y[x, d(x)] = 0 \]

\[ \Rightarrow d(c)y[x, d(x)] = 0, \text{ for all } x, y \in U \]

Since \( 0 \neq d(c) \in \mathbb{Z} \) and \( U \) is a left ideal of \( R \), then we have, \([x, d(x)] = 0, \text{ for all } x \in U. \)

By using the similar procedure as in Theorem: 1, then, we get, either \([z, x] = 0 \) (or) \( d(x) = 0. \)

Since \( d \) is non-zero, then \([z, x] = 0. \)

Hence \( R \) is commutative.

**Theorem 3**

Let \( R \) be a prime ring with \( \text{char} \neq 2, \) \( U \) a left ideal of \( R \) and \( d \) be a non-zero right reverse derivation of \( R. \)

If \([d(y), d(x)] = 0, \text{ for all } x, y \in U, \) then \( R \) is commutative.

**Proof**

Given that \([d(y), d(x)] = 0, \text{ for all } x, y \in U. \)

By taking \( yx \) instead of \( y \) in the hypothesis, then we get,

\[ \Rightarrow [d(yx), d(x)] = 0 \]

\[ \Rightarrow [(d(x)y + d(y)x), d(x)] = 0 \]

\[ \Rightarrow [d(x)y, d(x)] + [d(y)x, d(x)] = 0 \]

\[ \Rightarrow d(x)[y, d(x)] + [d(x),d(x)]y + d(y)[x, d(x)] + [d(y),d(x)]x = 0 \]

\[ \Rightarrow d(x)[y, d(x)] + [d(y),d(x)]x = 0, \text{ for all } x, y \in U \]

The proof is now completed by using equation (3) of Theorem: 2.

Hence \( R \) is commutative.

**REFERENCES**

