THE RELATIONSHIPS BETWEEN THE TRADITIONAL BELIEFS AND PRACTICE OF MATHEMATICS TEACHERS AND THEIR STUDENTS' ACHIEVEMENTS IN DOING MATHEMATICS TASKS

Aļesja Šapkova
Daugavpils University, Daugavpils, Latvia
E-mail: alesja.shapkova@gmail.com

Abstract

The importance of teachers' beliefs for students' learning is highlighted from different sources showing that teachers' beliefs affect their teaching approach that, in turn, affects students' achievement. The studies of mathematics teachers' beliefs in Latvia brought out a contradiction between teachers' constructivist beliefs on teaching and learning and their traditional routine work while revealing match between some traditional beliefs on teaching and learning and traditionally-oriented instructional practice. The aim of the present study is to explore the possible relationships between the traditional beliefs and practice of mathematics teachers and their students' achievement in mathematics. For this purpose the Latvian data from two international research projects was analyzed: Singapore National Education Institute project "Non-cognitive skills and Singapore learners – international comparison" and project "Nordic-Baltic comparative research in mathematics education" (NorBa). The sample included 190 mathematics teachers and their 2828 students from grade 9 representing different regions of Latvia, schools with different programs of education, rural areas and cities. The results suggest that the traditional beliefs of teachers are connected with lower students' achievement in mathematics test, while teachers' traditionally oriented self-reported practice is positively related to the achievement of their students. The research will discuss the reasons for the outcomes of this study that may refer to a certain extent to other European countries in the sphere of mathematics education that are currently trying to introduce reforms in their systems of education, as well as countries where students demonstrated lower achievements than OECD average in the mathematics test of PISA 2009.
Key words: *reported practice, students' achievement, teachers' espoused beliefs, teaching of mathematics.*

Introduction

PISA 2003 assessment data showed that such self-belief constructs as self-efficacy, self-concept, as well as such emotion as anxiety – are the best predictors of achievement in mathematics (OECD, 2004). Other studies have shown that students' mathematics confidence is the single most important predictor of math accuracy both within each country and pan-culturally, and accounts for most of the variance explained by the other self-constructs combined (Morony et all., 2013).

However, all the above-mentioned predictors are concerned only with learners and ignore the importance of teachers for students' learning. In studies with different predictors that may predict learners' achievements in mathematics, the teacher has not been studied as the most important single factor influencing students' achievements. The teacher's influence on learners' achievements may be regarded from different aspects, but the present study focuses on *the relatedness* of both teachers' traditional beliefs on teaching and learning in general (as well as teaching and learning *mathematics* in particular) and teachers' routine work in class to learners' achievements in doing mathematics tasks.

Theoretical Framework

Mathematics Teachers' Beliefs, and Their Relationship to Teachers' Pedagogic Practice

Belief entails individual, seldom – stable subjective knowledge that includes a person's feelings or care. Conscious change of beliefs may not always be predicted by logical judgement (Pehkonen, 1994). The given study focuses on teachers' beliefs. This notion (teachers' beliefs) is usually used to study teachers' educational beliefs (Borg, 2001). Teachers' beliefs are effected by the subjects they teach. Studies of teachers' beliefs in *mathematics education* have investigated teachers' beliefs about the *nature of mathematics* (Ernest 1989a), beliefs about *mathematics teaching* (Cobb et all., 1992), beliefs about the *learning of mathematics* (Ernst, 1989; Papert 1980) and beliefs about *students as learners* (van der Sandt, 2007; Tatto et all., 2012). This research will be focused on mathematics teachers' *beliefs on effective teaching and learning in general and beliefs on effective teaching and learning of mathematics* as beliefs about the way (mathematics) teaching and learning should occur ideally.

The world educational research manifested a dichotomous division of teachers' beliefs and approaches to teaching and learning: *the traditional beliefs* and *the constructivist beliefs*. The traditional beliefs or, in other words, a direct transmission view of student learning implies that a teacher's role is to communicate knowledge in a clear and structured way, to explain the correct solutions, to give learners clear and resolvable problems, and to ensure peace and concentration in the classroom (OECD, 2009). The *transmission orientation views mathematics* as a series of 'rules and truths' that must be conveyed to students and teaching as 'chalk and talk' followed by individual practice until fluency is attained (Swan, 2006).

The constructivist beliefs focus on students as active participants in the process of acquiring knowledge. Teachers holding these beliefs emphasize facilitating learners' inquiry. They prefer giving learners a chance to develop problem solutions on their own, and allow learners to play an active role in instructional activities (OECD, 2009). The *constructivist perception of mathematics* puts learners and their needs in the first place, and therefore emphasizes the use of learner-centered teaching methods (Dione, 1984).

However, it must be noted that the dichotomies so popular previously in the sphere of education are often criticized at present. Dichotomies such as teacher-centred versus student-centred classrooms, telling versus not-telling, speaking versus listening, and even teaching versus learning can restrict educators and educational theorists to a fragmented view of the classroom (Clark, 2006).

Philipp (2007) identified two orientations: 1) conceptual and 2) calculational to describe important dimensions on which teachers are known to differ. A teacher with a conceptual (cognitive-constructivist) orientation focuses strongly on concepts and holds that understanding is based on restructuring one's own prior knowledge (Staub and Stern, 2002). A conceptually oriented teacher is one whose actions are driven by an image of a system of ideas and ways of thinking she/he intends her/his students to develop; an image of how these ideas and ways of thinking can be developed; and an expectation and insistence that students will be intellectually engaged in tasks and activities (Philipp, 2007).

A teacher with a calculational orientation (direct-transmission) is more focused on acquiring basic numerical facts and mastering routines and procedures (Staub and Stern, 2002).

The actions of a teacher with a calculational orientation are driven by a fundamental image of mathematics as the application of calculations and procedures for deriving numerical results. Associated with a calculational orientation is a tendency to speak exclusively in the language of number and numerical operations, a predisposition to cast problem solving as producing a numerical solution, and a tendency to disregard context (Philipp, 2007).

This research will be singled out teachers' espoused beliefs from all the diverse other beliefs of teachers, because with the use of a questionnaire, as in the present research, can be reached only teachers' espoused conceptions. Espoused beliefs about teaching and learning could be defined as a teacher's subjective vision of effective, comfortable teaching adjusted to teacher and learners that may be implemented in practice or has been put to practice (Šapkova, 2012). They may act in their teaching practice totally differently. Two kinds of teacher's conceptions can be noticed, namely 'espoused' and 'enacted' (Ernest, 1989b). Studying teachers' espoused beliefs, there is always a possible threat to the validity of educational research based on the teachers' self-report, given that there may be inconsistencies between what teachers report, what they believe, and what they do (Bingimlas and Hanrahan, 2010). Without any prior preparation before the interview or survey, teachers often give socially desirable replies. But also teachers' espoused conceptions reflect their actual thinking, and therefore are worthwhile of study.

Review of Previous Research

Teachers' espoused beliefs are influenced by their actual beliefs (Thompson, 1992), by their knowledge and interpretation of advice about teaching problem solving (Fennema et all., 1989), and by their use and understanding of curriculum documents. In this respect it is important to pay attention, along with teachers' espoused beliefs, to their reported classroom practice. In the given research, exactly due to these reasons, the survey on the reported practice followed in a common package with the survey of teachers' beliefs. Teachers' self-reported practice (what is done) is teacher's recognition of methods used in class and the frequency of using them (for instance, almost every lesson, at about half of lessons, in some lessons, never) (apkova, 2012). However, it must be noticed that reported classroom practices are influenced not only by espoused beliefs, but also by actions in the classroom as well as by constraints such as a parent and student beliefs and school culture (Anderson, 1997).

The question of how teachers' beliefs concerning the learning and teaching of mathematics influence and relate to their practice is a highly active field of research (Philipp, 2007). Such urgency of studying teachers' beliefs is justified by the fact that, once acquired, beliefs are actualized during classroom instruction (Leder et all., 2002; Handal, 2003). Teachers' beliefs are like a filter affecting teachers' decisions more than their knowledge in education or academic programme guidelines (Clark and Peterson, 1986). Hence, teachers' beliefs affect the manner of teaching that, in turn, as proved in recent studies, have a great impact on learners' achievement (OECD, 2009).

But it must be taken into consideration that some studies report inconsistencies between teachers' beliefs and practices (Thompson, 1984; Fang, 1996). Although elementary school teachers often enter the teaching profession with nontraditional beliefs about how they should teach, when faced with the constraints of actual classroom teaching, they tend to implement more traditional classroom practices (Brown and Borko, 1992; Raymond, 1997).

Students' Achievements in Mathematics and Their Relatedness to Teachers' Beliefs and Teachers' Pedagogic Practice

Judgments about the effectiveness of teachers in bringing about the above learning outcomes may be based on expectations of and evidence about one or more of the following: 1) teacher behaviour; 2) pupil behaviour; 3) pupil learning outcomes (Askew et all., 1997). It was decided in this study that the identification of effective teachers would be based not on presumptions of 'good practice' but on the evidence of the level of their learners' achievements.

In this study, the broad concept of students' achievements was limited by students' achievement in doing mathematics tasks. Students' achievements doing mathematics tests could be defined by learners ability in computations and solving a specific and non-traditional mathematics problems, which can normally be measured by computer tests.

As noted previously, research evidence, although limited, suggests the following: 1) positive student outcomes are most likely to be associated with teachers who support the notions that mathematics is a process of enquiry and that learning mathematics requires an active involvement; 2) less likely to be associated with teachers who support the beliefs that mathematics is a set of rules and procedures, learning mathematics requires following the teacher's direction, and mathematics is a fixed ability (Tatto et all., 2012).

Staub and Stern (2002) compared achievement gains made by students taught by teachers holding a cognitive-constructivist orientation with those made by students whose teachers held a direct-transmission view. They found that the students whose teachers had a stronger cognitive constructivist orientation displayed higher achievement gains in dealing with mathematical problems than did students whose teachers had less of a cognitive constructivist view, subscribing instead to pedagogical content beliefs that are consistent with a direct-transmission view of learning and teaching.

On the other hand, there are also studies of a positive influence of the traditional beliefs and practice on learners' achievements. A classroom observation study of over 100 mathematics teachers revealed that the focus on efficient behaviors made it possible to distinguish effective from ineffective teachers, and that it was teacher, who spent more time teaching the whole class (traditional practice) as opposed to teaching individual learners, whose learners showed stronger gains in mathematics achievement (Muijs and Reynolds, 2001). Flores and Kaylor (2007) found that at risk students perform better, have higher student achievement and are more attentive in class when given direct instruction with traditional teaching methods. Close monitoring, adequate pacing and classroom management as well as clarity of presentation, well-structured lessons and informative and encouraging feedback – known as key aspects of "direct instruction"– have generally been shown to have a positive impact on student achievement (OECD 2009). However, notwithstanding the sufficiently large number of research projects on a positive impact of traditional practice on the learners' achievement and the numerous models indicating a direct relationship between beliefs and practices (Ernest, 1988; Clark and Peterson, 1986), there are no sufficient proofs for the fact that teachers' traditional beliefs would also have a positive impact on learners' achievement.

Any changes in education trigger discussions about the way these reforms and changes will affect learners' achievements and the most conservative subjects arouse the most red-hot discussions. Mathematics with its philosophy that often does not match the advanced currents in philosophy of education causes most questions: for instance, which of approaches – the traditional or the constructivist, the cognitive-constructivist or the direct-transmission – is most efficient in teaching mathematics, etc. These discussions continue and no single answer has been received yet. One reason may be due to the fact that many scholars criticize constructivism and the attempts to sketch the influence of constructivism in current mathematics and science education (for example, Matthews, 1993) suggesting that constructivist theories are misleading or contradict the previously made findings.

The conflicting results of previous studies, the conservative nature of the science of mathematics, criticism of constructivism were the starting point for choosing the *traditional (direct transmission) beliefs* of mathematics teachers for this study.

The Aim of the Study

Ponte and Chapman (2006) pointed out the need for researchers who continue to work in the area of teachers' beliefs to start combine these constructs (beliefs) with others related to practice in more creative ways. This research offers to combine teachers' beliefs with their students' achievement.

The aim of the present research is to make out whether there are correlations between the *traditional* orientation in *teachers' espoused beliefs* and self-reported practice and the achievements in doing mathematics tasks of the students taught by these teachers.

Proceeding from the above-mentioned considerations, two research questions were set: 1) How consistent are traditional espoused beliefs of Latvian teachers on teaching and learning and their students' achievement in doing mathematics tasks? 2) How consistent are the traditional orientation of Latvian teachers in self-reported practice and their students' achievement in doing mathematics tasks?

Methodology of Research

General Background of Research

Globalization, density of information on the one hand, and the outcomes of international comparative education research (Trends in International Mathematics and Science Study (TIMSS), Programme for International Student Assessment (PISA)) on the other, set forth the requirements for further improvement of the system of education in all countries including Latvia. Therefore, in 2006 and 2008 new standards in basic (ISCED level 1) and secondary (ISCED level 2) education were introduced in Latvian system of education, also in mathematics education. These reforms changed the philosophy of Latvian mathematics and science education system by introducing the fundamental principles of constructivism: goals were set to provide learners with basic knowledge and skills needed for their public and personal life, to form the basis of learners' further education, facilitate their harmonious development and responsible attitude towards oneself, family, society, surrounding environment, and country (Latvian National Standards, 2006; 2008). The authors of this reform implicitly hope that teachers in Latvia 1) will integrate the constructivist ideas of this reform in their belief system 2) will implement these constructivist beliefs in their daily work and, in its turn, 3) this constructivist practice will improve learners' achievements.

To support effective curriculum implementation, it is essential to change the teacher beliefs. Prior studies of Latvian teachers' beliefs revealed that the espoused beliefs of Latvian teachers of mathematics on efficient and effective (mathematics) teaching and learning are more tended to a *constructivist* approach, though in many cases teachers still hold strong *traditional* and *formally* tended beliefs. At the same time the reported classroom practices of Latvian teachers are more oriented to a *traditional* approach; yet there exist statistically significant differences for teachers of different social and demographic groups (Pipere, 2005; Šapkova, 2011; 2012). Thus, on the one hand, there is an obvious contradiction between the *constructivist* beliefs of Latvian teachers and their *traditional* routine practice. On the other hand, there is the coincidence in some *traditional* beliefs of mathematics teachers on teaching and learning and in their *traditional* routine practice.

Despite the fact that for several years Latvian national standard of mathematics has fully matched the content of mathematical sums used in PISA tests until now, the analysis of the results of PISA 2009 showed that the average achievement of Latvian school learners in this period were considerably lower than those of OECD countries. Among all 65 member countries, Latvia ranged no higher than 32. Only in some European countries the achievements were worse (Greece, Romania, Bulgaria, Azerbaijan, the Balkan countries). Comparison of competence groups revealed that there are still very few learners in Latvia whose knowledge of mathematics would correspond to the highest level of achievement (Geske et all., 2010; apkova, 2013). PISA research is focused on 15 years old learners. The low achievements of 15 years old Latvian learners in PISA research motivated the choice learners of the grade 9 for this research.

Sample Selection

The sample included 190 mathematics teachers of grades 7-9 and their 2828 grade 9 students from different regions of Latvia, from rural areas and cities, from schools with different education programs (see Table 1).

The teacher sample was formed by teachers of different age (ranged from 26 to 66 years, M=46, the dominating age group from 40 to 49 (45.3%)), level of education (the majority of respondents hold bachelor (54.2%) or master's (43.7%) academic degree as well as work experience (ranged from 3 to 44 years, M=23, the dominant group from 26 to 30 years (24.2%)). 97.4% of all teachers were female and 2.6% were male.

From all students 51.8% were female and 48.2% were male, 79.2% studied in schools of general education, and 20.8% studied in school of ethnic minorities, 43.3% lived in big towns, and 56.7% lived in small town or countryside (see Table 1).

Table 1. Distribution of teacher and student samples into socio-demographic groups.

Socio-demographic indicators		Teachers' sample frequency N (%)	Student's sample frequency N (%)
Region of Latvia	Riga (capital) and region	71 (37.4)	1304 (46)
	Latgale	41 (21.6)	482 (17)
	Vidzeme	31(16.3)	369 (13)
	Kurzeme	24 (12.6)	327 (12)
	Zemgale	23(12.1)	346 (12)
Sex	Female	185 (97.4)	1464(51.8)
	Male	5 (2.6)	1364(48.2)
Education* programme	General	151 (79.5)	2241 (79.2)
	Ethnic minority	39 (20.5)	587 (20.8)
Urbanization	Urban	66 (34.7)	1225 (43.3)
	Rural/small town	124 (65.3)	1603 (56.7)

*Instruction for ethnic minority learners is manifested in both teaching different academic subjects in different languages in one education establishment for the same group of learners and using two languages in explicating the content of an academic subject in the same class if the language of instruction is not the native language of the learners. Instruction in schools with general education programme is only in the Latvian language.

Instrument and Procedures

Data of two international research projects were used in the present research: Singapore National Education Institute project *"Non-cognitive skills and Singapore learners – international comparison"* and project "*Nordic-Baltic comparative research in mathematics education*" (NorBa).

Within NorBa project, teachers' survey was elaborated by project leaders (Lepik and Pipere 2011) with the aim of making out Latvian mathematics teachers' beliefs on teaching mathematics. This study was based on 3 parts of the quantitative survey of NorBa project: D, E, and G (50 statements in total). Parts D (Teacher's beliefs on efficient/good teaching) and E (Teachers' beliefs on efficient/good teaching and learning mathematics) were evaluated according to 5 point Likert scale from 'strongly agree' to 'strongly disagree'. Part G (Teacher's daily instructional practice in class) was evaluated according to 4 point scale: almost every lesson, at about half of lessons, in some lessons, never. Formulations of the survey items will be shown in Table 2 within Results section.

Within Singapore National Education Institute project, the students' survey was elaborated to compare the personality features of students, their attitude towards mathematics and their abilities to do mathematical sums in several countries (Morony et all., 2013).

This research is based on the part of projects' survey containing 12 tasks of mathematics. All tasks were multiple choice or short reply tasks and were drawn from PISA or TIMSS issues of recent years (for more information see Table 2). Learners received one point for each correctly done task. If the task was not done or was not done correctly, student's achievements were evaluated as 0. Exceptions are tasks 2, 7, and 11. Each of these tasks contained four subtasks, thus for doing each of these tasks learner could receive from 0 to 4 points. The Cronbach Alpha coefficient for this part of the survey was 0.582. Thus, it may be concluded that reliability level is average and may be used in the research, though the results will not have the highest degree of probability yet. One reason for the lower reliabilities for the mathematics test is the small number of items relative to the breadth of the construct – by contrast, PISA 2003 had 85 mathematics items (Morony et all., 2013). Yet, it was observed that, if items 2 and 11 are deleted, Cronbach's alpha grows and reaches the value 0.615. Hence, only 10 tasks described below were used for cluster analysis (from 1 to 12, except for 2 and 11).

The data collection process took place in October - December 2010, in several stages. Teachers who at that moment worked in grade 9 were sent an invitation to participate in mathematics teachers' survey as well as organize research in the grade 9 they were teaching. Those teachers who agreed to organize the research and participate in the survey were given ID number and sent instructions and teacher questionnaires by e-mail. Respondents sent the filled in questionnaires electronically to an e-mail address specially made for this purpose.

Learners had to fill in the survey questionnaires electronically, in Russian or Latvian by choice, opening an Internet address specially designed for this purpose. Therefore, each learner needed a computer with Internet connection as well as some notepaper sheets for draft copies of mathematics tasks calculations. When doing mathematics tasks, learners were also allowed to use calculators. If learners could not do any of the tasks, they were allowed to skip them.

In general, students' survey was organized in computer labs and was guided by help of mathematics or other teachers who instructed the students in the course of completing the survey. Students had to work independently, without consulting their teacher or classmates, relying only on their own experience and knowledge. Though whole classes of students were invited to participate in the survey, it was not demanded that all students from the respective class take part in it. Participation in this research was voluntary, the principle of confidentiality being attributed to respondents' identity and research notes.

It must be noted that the number of students and teachers who participated in two previously mentioned studies was much greater, yet not all students and teachers were later selected for further data analysis. Students' data were connected to those of their teachers by means of teacher's ID number: each student's replies were scored on his/her teacher's account. From 3083 students who participated in the study 2828 students were selected. They were not selected for future research if their teacher's ID number was erroneous or was not indicated, or if their teacher was not a participant of NorBa research.

Data Analysis

The following methods of statistical analysis were used for data processing: Kolmogorov-Smirnov test to assess the distribution of data, descriptive statistics, frequencies, two step cluster analysis, factor analysis, hi-quadrangle criterion, Wilcoxon criterion, Kruskal-Wallis criterion, Mann-Whitney criterion as well as Cronbach Alpha to assess the reliability.

According to Kolmogorov-Smirnov criterion, the data on all indicators significantly differed from normal distribution ($p<0.01$), therefore the statistical analysis of differences in items was conducted using non-parametrical criteria of data analysis.

Results of Research

Each part D, E and G of teacher survey were included in the factor analysis with the aim of singling out the factors of traditional teaching. Each part had the value of KMO above 0.50, that means that the data were valid for factor analysis. The Principal Component Analysis with Varimax rotation and Kaiser normalization located traditional teaching factors with the eigenvalue greater than 1. Cronbach's Alpha value was determined for each factor of traditional teaching (see Table 2) and new summarized indicators *D_trad, E_trad un G_trad* was formed: the traditional factor replies in each part were summed up for each respondent.

Table 2. The results of factor analysis for teachers' survey.

		KMO value	Traditional teaching items	Cronbach's Alpha
Espoused beliefs	D part	0.746	D2 Instruction should be built around problems with clear, correct answers, and around ideas that most learners can grasp quickly. D3 How much learners learn depends on how much background knowledge they have – that is why teaching facts is so necessary. D4 Good teachers demonstrate the correct way to solve a problem D16 A quiet classroom is generally needed for efficient learning.	0.683
	E part	0.753	E6. In mathematics teaching, one has to practice much above all. E13. The learning of central computing techniques must be stressed. E14. Pupils should get the right answer when solving tasks. E17. Routine tasks should be solved where the use of the known procedure will surely lead to the result. E19. Mathematical knowledge, such as facts and results, should be taught. E11. In particular, the use of mathematical symbols should be practiced	0.696
Reported practice	G part	0.620	G1 Memorize formulas and procedures G2 Apply facts, concepts and procedures to solve routine problems	0.520

Besides, for further data analysis new aggregated indicators ED_trad and EDG_trad were used: indicators E_trad, D_trad and G_trad were summed up for each respondent.

The results acquired in the mathematics test were also transformed into a single new aggregated indicator *Problem_total*: the twenty one factor replies in mathematics test were summed up for each respondent (see Table 3). 3 sums (2, 7, and 11) out of 12 tasks of the mathematics test entailed 4 subtasks, therefore in the formation of the new summarized indicator the solution of 21 mathematics tasks/subtasks was taken into consideration: 9 test sums without subtasks and 12=3x4 subtasks of the rest of three tasks. Cronbach's Alpha for 21 mathematics sums was 0.675.

Table 3. Characteristics of learners' mathematics test tasks.

Task no.	Task content	Task mathematical content	Task competence group	Task survey, source, item code
1.	Coloured Candies	Probability	I	PISA 2003, M467Q01
2.	Carpenter	Space and form	II	PISA 2000, M266Q01
3.	Bookshelves	Numbers and measures	II	PISA 2003, M484Q01
4.	Choices	Combinatorics	I	PISA 2003, M510Q01
5.	Science Tests	Statistics	II	PISA 2003, M468Q01
6.	Patio	Numbers and measures	I	PISA, field trial, M267Q01
7.	Drug Concentrations	Variables and functional correlations	II	PISA, field trial, M307Q01
8.	Space Flight	Numbers and measures	II	PISA, field trial, M543Q01
9.	Earthquake	Probability	III	PISA 2003, M509Q01
10.	Zedland	Variables and functional correlations	I	TIMSS 2006, pg. 106
11.	Payment by Area	Numbers and measures	III	PISA 2003, M480Q01
12.	Skateboard	Combinatorics	I	PISA 2003, M520Q02

On the basis of Cronbach Alpha values there was concluded that the reliability level is average and all parts of survey may be used in the research, though the results will not have the highest degree of probability yet for part G.

The analysis of the correlation between the traditional beliefs and practice of mathematics teachers (indicators *D_trad, E_trad, G_trad*, as well as indicator sum *ED_trad, EDG_trad*) and their students' achievements (indicator *Problem_total*) shows that all indicators (both single and aggregated) of teachers' beliefs and classroom practice have small, yet statistically significant correlations with their students' achievements (see Table 4). As there is still an opportunity that the correlation of two indicators has been determined by the influence of a third indicator (for instance, a socially demographical indicator that is common both for teachers and learners), partial correlation was also conducted thus controlling the impact of urbanization, school academic programme, and region.

Table 4. Intercorrelations of the teachers' beliefs and practice and students achievements (Spearman's correlation).

	Indicators of teachers' orientation toward traditional teaching and learning	Indicator of students' achievement: Correlation coef-ficient	Indicator of students' achievement: Partial correlation (control variables: urbanization & academic programme & region)
Espoused beliefs	D_trad	-0.038**	-0.040*
	E_trad	-0.091**	-0.091**
	ED_trad	-0.094**	-0.074**
Reported practice	G_trad	0.091**	0.103**
Espoused beliefs & reported practice	DEG_trad	-0.059**	-0.059**

* p<0.05; ** p<0.01

As appears from Table 4, there is a significant positive correlation (p<0.01) between teachers' traditional self-reported practice and their students' achievements in doing 12 tasks, while all correlations among indicators that include teachers' traditional beliefs and learners' achievements are statistically significant and negative (p<0.01). The significance, power and direction of the correlation coefficient in a partial correlation, controlling urbanization, school academic programme and Latvia regions, practically do not change, that indirectly testifies to the fact that there exists a stable correlation between teachers' traditional beliefs and their students' achievements in doing mathematics tasks.

To verify the correlation between teachers' traditional beliefs, practice and students' achievements, Wilcoxon criterion was used only for those indicators for whom statistically significant correlation was located. Wilcoxon criterion also shows that there exists coherence between the same indicators of teachers' beliefs and their students' achievements (p<0.01) (see Table 5).

Table 5. Coherence between Latvian mathematics teachers' traditional beliefs, practice and their students' achievements.

		N	Mean Rank	Sum of Ranks
Problem_total - E_trad	Negative Ranks	2712[a]	1412.79	3831473.50
	Positive Ranks	58[b]	109.68	6361.50
	Ties	21[c]		
	Total	2791		
Problem_total - ED_trad	Negative Ranks	2791[d]	1396.00	3896236.00
	Positive Ranks	0[e]	0.00	0.00
	Ties	0[f]		
	Total	2791		
Problem_total - EDG_trad	Negative Ranks	2791[g]	1396.00	3896236.00
	Positive Ranks	0[h]	0.00	0.00
	Ties	0[i]		
	Total	2791		
Problem_total - D_trad	Negative Ranks	2252[j]	1457.39	3282031.50
	Positive Ranks	437[k]	765.84	334673.50
	Ties	139[l]		
	Total	2828		
Problem_total - G_trad	Negative Ranks	103[m]	301.39	31043.00
	Positive Ranks	2603[n]	1395.13	3631528.00
	Ties	122[o]		
	Total	2828		

a. Problem_total < E_trad	f. Problem_total = ED_trad	k. Problem_total> D_trad
b. Problem_total > E_trad	g. Problem_total < EDG_trad	l. Problem_total= D_trad
c. Problem_total = E_trad	h. Problem_total > EDG_trad	m. Problem_total< G_trad
d. Problem_total < ED_trad	i. Problem_total= EDG_trad	n. Problem_total> G_trad
e. Problem_total > ED_trad	j. Problem_total< D_trad	o. Problem_total= G_trad

Thus, like in the case of correlation analysis, it appears that the more distinct teacher's traditional beliefs (indicators E_trad, ED_trad, DEG_trad) match the lower achievement of his/her students. However, the self-reports of a more frequent use of the traditional way of instruction in mathematics match the higher achievement of students (indicator G_trad).

This outcome could have been influenced by the fact that indicator G consists of just two items, while indicator D consists of four items.

Cluster analysis was made with the aim of confirming the data of correlation analysis and examining students' achievements in relation to concrete scale items. As has been mentioned before, for cluster analysis only 10 items described above were used (from 1 to 12, except 2 and 11). Using two-step cluster analysis, all learners were divided into four clusters depending on their replies to 10 questions (see Table 6).

Table 6. The number of learners (%) having done the particular task correctly in each cluster.

	The number of learners (%) having done the particular task correctly in each cluster										
Task no. (predictor importance)	1. (0.64)	3. (1.00)	4. (0.18)	5. (0.30)	6. (0.34)	7.* (0.57)	8. (0.23)	9. (0.14)	10. (0.47)	12. (0.41)	Total
Cluster 1 (N_1=957, 33.8%)	30.1	00.0	30.8	20.1	8.5	10.4	39.7	37.8	49.6	18.8	1-8
Cluster 2 (N_2=386 13.6%)	00.0	00.0	00.0	12.7	0.00	0.00	38.1	31.9	0.00	0.00	0-3
Cluster 3 (N_3=553 19.6%)	35.2	86.6	47.9	64.4	46.7	55.9	79.9	68.0	85.7	60.0	5-12
Cluster 4 (N_4=932, 33.0%)	22.1	100.0	26.4	23.7	14.6	12.2	34.2	32.9	45.7	14.8	1-8

* The number of students who received just 3 or 4 points (%) was indicated

Comparing four clusters, cluster 2 comprises the students with the lowest achievements because none of them did tasks 1-4 as well as 6, 7 10 and 12, except for 5, 8, and 9. Hence, the total achievement of students in cluster 2 will be from 0 to 3 points. In turn, the highest achievements in doing 10 tasks were demonstrated by the students from cluster 3: in this cluster the percentage of the students who did all tasks correctly (except task 3) is the highest. The number of points scored by the students for 10 tasks is between 5 and 12.

The main differences of the students from clusters 1 and 4 appear in the results of task 3: none of the students from cluster 1 did this task, while all students from cluster 4 (100%) managed to do it (see Table 6). Besides, in cluster 1, 67.5% of students scored up to 3 points for all 10 tasks, but in cluster 4 the number of such students is smaller – 45.8%. The number of points scored by the students for 10 tasks in both clusters lies within 1 to 8.

To determine whether there exists a statistically significant difference between the traditional beliefs of the teachers of students of four clusters, each item (see Table 2) was cross-tabulated considering students' clusters (see Table 6). By means of the chi- square test it was clarified that there exist statistically significant differences in teachers' replies: teachers' replies do not have equal frequency/equal distribution in traditionally oriented items D2, D4, D16, E6, E11, E13, E17 and G1 ($p<0.01$). Teachers whose students belong to cluster 3 (those with the highest achievements), in comparison to those teachers whose students belong to other clusters, hold traditional statements on teaching and learning least (except for statement D4: *Good teachers demonstrate the correct way to solve a problem*) (see Table 7 and Table 8). At the same time these teachers more often than others use traditional methods – memorizing formulas and procedures.

Kruskal-Wallis criterion also showed that there exist statistically significant differences in teachers' replies to questions D4, D16, E13 and G1 ($p<0.01$), also it is possible to talk about tendencies in questions E11 and E17 ($p=0.08$) (see Table 7 and Table 8).

Table 7. Descriptive statistics on Latvian mathematics teachers' traditional espoused beliefs, cross-tabulated considering four students' clusters.

Items	Clusters of learners	Teachers' replies (%)		
		Don't agree	Neutral	Agree
D2	1.	20.6	31.9	47.5
	2.	21.8	30.8	47.4
	3.	18.1	43.9	**40.0**
	4.	24.5	31.2	44.2
D4	1.	30.0	38.6	31.4
	2.	31.6	40.9	27.5
	3.	24.3	32.5	43.3
	4.	34.9	32.6	32.5
D16	1.	43.3	35.5	21.2
	2.	23.3	29.8	46.9
	3.	26.1	38.9	**35.1**
	4.	40.4	33.0	26.6
E6	1.	5.5	30.3	64.2
	2.	3.5	28.8	67.6
	3.	10.0	28.9	**60.1**
	4.	4.7	29.5	65.9
E11	1.	14.6	37.7	47.6
	2.	15.0	38.1	46.9
	3.	9.9	51.7	**38.3**
	4.	13.7	37.8	48.5
E13	1.	1.1	17.3	81.5
	2.	0.8	15.5	83.5
	3.	1.8	23.7	**64.1**
	4.	2.0	15.9	82.0
E17	1.	33.2	39.6	27.2
	2.	30.1	42.4	27.5
	3.	39.4	38.3	**22.2**
	4.	35.1	34.8	30.0

Table 8. Descriptive statistics on Latvian mathematics teachers' traditional reported practices, cross-tabulated considering four students' clusters.

Items	Clusters of learners	Teachers' replies (%)			
		Never	Some	Half	Each lesson
G1	1.	24.8	53.7	19.4	2.1
	2.	25.1	58.0	16.3	0.5
	3.	20.4	47.0	30.2	2.4
	4.	21.6	58.2	18.6	1.7

To understand whether there exist statistically significant differences in replies of teachers whose learners had the highest/lowest achievements in mathematics test, clusters 2 and 3 should be under the scrutiny. For this purpose Mann-Whitney criterion was used. Statistically significant differences were revealed in teachers' replies to items D4, D16 and G1 ($p<0.01$), items E13, E17 ($p<0.05$), besides it is possible to discern the following tendencies in items E6 ($p=0.056$) and E14 ($p=0.055$): teachers whose students belong to cluster 3, in comparison to those whose students belong to cluster 2, less agree to traditional statements about teaching and learning, except for statement D4.

Discussion

The given research is a local study in an EU post socialist country, yet the results of it may refer to a certain extent to some other European countries in the sphere of mathematics education that are currently trying to introduce reforms in their systems of education, as well as countries where students in the mathematics test of PISA 2009, like students in Latvia, demonstrated lower achievements than OECD average.

The research database shows that there exists a correlation between Latvian teachers' espoused traditional beliefs on teaching mathematics and their students' achievements in doing mathematics tasks: the more distinct teachers' espoused traditional beliefs, the lower their students' achievements. This is proved by the negative correlation coefficients, Wilcoxon criterion based on a negative rank, as well as the results of cluster analysis. Cluster analysis showed that teachers whose students demonstrated the highest achievement in solving 10 mathematics tasks express the least agreement with traditional ideas on teaching and learning as compared to teachers whose students belong to other clusters. These results indirectly confirm the outcomes of other studies as well. For instance, Askew, Brown, Rhodes, Johnson, Wiliam (1997) found that highly effective teachers were characterized by connectionist beliefs, while *transmission* and discovery orientations tended to characterize some *of the less effective teachers*.

Low correlation coefficient (approx. 0.1) in this research may be accounted for by the fact that teachers of mathematics in Latvia are more tended towards constructivism in their espoused beliefs on approaches to teaching and learning (Šapkova, 2011; 2012). Thus the sampling of the teachers' research did not demonstrate any *distinct* traditional espoused beliefs about mathematics teaching and learning.

On the other hand, the results show that teachers' traditional inclinations in their reported practice has a positive impact on their students' achievements. This, in turn, is testified both by the positive correlation coefficients and Wilcoxon criterion based on positive rank. Cluster analysis shows that teachers whose students demonstrated the highest achievements in doing 10 mathematics tasks, as compared to teachers whose students belong to other clusters, in their reported practice make students memorize formulas and procedures.

The prior research demonstrated that, despite the fact that Latvian mathematics teachers' beliefs on teaching are more tended towards constructivism, in their routine work teachers almost each lesson use traditional learning approaches (apkova, 2011; 2012). A number of studies report inconsistencies between what teachers believed and what they actually did in the classroom (Fang, 1996). Beliefs about mathematics teaching and learning could be less traditional then actual teaching practices (Raymond, 1997).

This is not entirely surprising, and can happen for a number of reasons. The first is that teachers' individual beliefs may not be the same as those held by their head teachers or the government, so that they may be forced to use approaches that they themselves would not feel are the most effective (Muijs and Reynolds, 2001). Indeed, as mentioned before, using some elements of the traditional approach in everyday practice is justified as it gives an opportunity for Latvian teachers to reach better results in teaching Latvian school learners. Moreover, classroom practices in Latvia are often influenced by parents' and learners' beliefs and school culture (Anderson, 1997).

Lāce (2010), having analyzed Latvian teachers' statements in interviews and their self-evaluation after observed lessons, concluded that Latvian primary school mathematics teachers have little experience of analyzing their work. This might be the reason why the constructivist approach has not been widely implemented in teachers' routine work yet. Indeed, the research literature suggests that changing the mathematics teachers' beliefs toward the constructivist orientation necessitates professional development in mathematics that has two distinct components (Phillip 2007). First, teachers must be challenged to reflect on their existing beliefs systems for changes to occur. Second, teachers must develop an understanding of the benefits of a constructivist perspective on teaching and learning on learners' achievement for changes to take place.

Item D4 (*Good teachers demonstrate the correct way to solve a problem*) may be considered as a part of teachers' reported traditional practice. Teachers, replying to this item, could express not their espoused beliefs, but self-reported practice, because they identified themselves as good teachers. This may account for similar tendencies in replies to items G1 and D4. Besides, recently a hypothesis has been expressed that a great part of learners do not wish or cannot perform well after constructivist learning process and teachers are well aware of this. In this case a part of Latvian school learners may work only according to an example that must be provided by a good teacher.

Completely different tendencies of teachers' espoused beliefs and reported practice may be accounted for by the fact that the major factor of influence on learners' achievement is not the teachers' beliefs but their readiness to change, i.e. to change their beliefs as well as an ability to adapt their practice to the learners' intellectual level, needs, and motivation. At the same time these teachers continue to use traditional methods gradually introducing new ones in their work.

Despite the fact that the education philosophy reflected in the State Education standard is oriented toward the process of learning, in reality the mathematics education in Latvia is measured by learners' achievement that, in turn, is characteristic of the traditional paradigm of education. Indeed, after every three years every learner in Latvia must take a compulsory centralized test or state examination in mathematics, regular mathematics Olympiads have been organized on local and state level. Learners' achievements in Olympiads and examinations are the basis of mathematics teacher ratings. This may account for the great influence of traditional reported practice on learners' achievements, because teachers believed that students would have higher mathematical achievement when using the traditional methods of teaching mathematics (Jennings and Pawat, 1997). Besides, the broad social and cultural climate of the classroom may influence teachers' espoused beliefs on mathematics, mathematics learning, teaching and assessment (Barakatas, 2005).

Hence, the question, whether the traditional approach is to be evaluated positively or critically, may be answered only with regard of the local and global context and the desirable goal set. It seems that constructivism is still applicable to the process of learning and knowledge in long term, but traditionalism, possibly, suits better the production of an instant product and provides for achieving better short-term results in tests.

The research has several limitations: first, only grade 9 learners and their teachers participated in the research. Learners did mathematics tasks electronically and in a short time limit, therefore successful completion of the test may have been related to how well certain procedures and formulas had been memorized, and also test anxiety could have had a certain impact. Moreover the teachers involved in the survey were the actual teachers of the students, but it was not known how long they have been teaching them.

Conclusions

The results of this research suggest that the traditional beliefs of teachers are connected with lower students' achievement in mathematics test, while teachers' traditionally oriented self-reported practice is positively related to the achievement of their students.

The outcomes of this study may be used for the development and improvement of courses in mathematics teacher education: teacher further education must also focus on the inner readiness of teachers to accept reforms, not just acquiring certain skills and abilities. This will give an opportunity to adjust both the further education programme for experienced teachers and the training programme for becoming teachers in higher education establishments with the aim of both securing the efficiency of the education reform and not losing the gains of the preceding stage of education reform as well as finding reasons why the seemingly well-meant and well-organized education reforms do not always yield the expected outcomes.

In further studies it may be verified whether teachers' other beliefs on teaching mathematics affect students' achievements. It is possible that a mixture of different beliefs (traditional, formal, and constructivist) negatively affects learners' achievements.

References

Anderson, J. (1997). Teachers' reported use of problem solving teaching strategies in primary mathematics classrooms. In F. Biddulph & K. Carr (Eds), *People in mathematics education* (pp. 50-57). Rotorua, NZ: MERGA.

Askew, M., Brown, M., Rhodes, V., Johnson, D., & Wiliam, D. (1997). *Effective Teachers of Numeracy (Final Report)*. London: King's College.

Barakatas, A. (2005). A typology of mathematics teachers' beliefs about teaching and learning mathematics and instructional practices. *Mathematics Educational Research Journal*, *17* (2), 69-90.

Bingimals, K., & Hanrahan, M. (2010). The relationship between teachers' beliefs and their practice: How the literature can inform science education reformer and researchers. In M.F. Tasar & G. Cakmakci (Eds.), *Contemporary Science Education Research: International perspectives* (pp. 415-422). Ankara, Turkey: Pegem Akademy.

Borg, M. (2001). Teachers' beliefs. *ELT Journal*, *55* (2), 186-188.

Brown, C. & Borko, H. (1992). Becoming a mathematics teacher. In D. Grouws (Ed.) *Handbook of research on mathematics teaching and learning* (pp. 209-242). New York: Macmillan Publishing Company.

Clarke, D.J. (2006). Using International Comparative Research to Contest Prevalent Oppositional Dichotomies. Zentralblatt für Didaktik der Mathematik, 38 (5), 376-387.

Clark, C. M. & Peterson, P. L. (1986). Teachers' thought processes. In M. C. Wittrock (Ed.), *Handbook of research on teaching* (pp. 255-296). New York: Macmillan.

Cobb, P., Wood, T., & Yackel, E. (1992). A constructivist approach to second grade mathematics. In E. Von Cooney, T.J. (1985). A beginning teacher's view of problem solving. *Journal for Research in Mathematics Education*, *16* (5), 324-336.

Dionne, J. (1984). The perception of mathematics among elementary school teachers. In J. Moser (Ed.), *Proceedings of the 6th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 223–228). Madison, WI: University of Wisconsin.

Ernest, P. (1988). *The Impact of Beliefs on the Teaching of Mathematics*. Paper presented at the 6th International Congress of Mathematical Education, Budapest, August.

Ernest, P. (1989a). The knowledge, beliefs and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching*, *15*, 13–34.

Ernest, P. (1989b). The Impact of Beliefs on the Teaching of Mathematics. In P. Ernest, Ed. *Mathematics Teaching: The State of the Art*. London: The Falmer Press.

Fang, Z. (1996). A review of research on teacher beliefs and practices. *Educational Research*, *38* (1), 47-65.

Fennema, E., Carpenter, T. P., & Peterson, P. L. (1989). Teachers' decision making and cognitively guided instruction: A new paradigm for curriculum development. In N. F. Ellerton & M. A. Clements (Eds.), *School mathematics: The challenge to change* (pp. 174–187). Geelong, Victoria, Australia: Deakin University Press.

Flores, M., & Kaylor, M. (2007). The effects of a direct instruction program on the fraction performance of middle school students at-risk for failure in mathematics. *Journal of Instructional Psychology, 34* (2), 84-94.

Geske A., Grīnfelds, A., Kangro, A., & Kiseļova, R. (2010). *Ko skolēni zina un prot – kompetence lasīšanā, matemātikā un dabaszinātnēs. Latvija OECD valstu Starptautiskajā skolēnu novērtēšanas programmā 2009.* Rīga.: Latvijas Universitāte.

Jennings, P., & Prawat, R. (1997). Students as context in mathematics reform: The story of two upper elementary teachers. *The Elementary School Journal, 97* (3), 251-270.

Latvian National Standards (2006). Noteikumi par valsts standartu pamatizglītībā un pamatizglītības mācību priekšmetu standartiem. Resource document. http://www.likumi.lv/doc.php?id=150407. Accessed 25 April 2013.

Latvian National Standards (2008). Noteikumi par valsts vispārējās vidējās izglītības standartu un vispārējās vidējās izglītības mācību priekšmetu standartiem. Resource document. http://www.likumi.lv/doc.php?id=181216. Accessed 25 April 2013.

Lāce, G. (2010). Latvijas pamatskolas matemātikas skolotāju competence matemātikas didaktikā. Resource document. http://www.lu.lv/fileadmin/user_upload/lu_portal/zinas/kopsavilkums_latv_LaceG.pdf. Accessed 5 May 2011.

Leder, G. C., Pehkonen, E., & Törner, G. (Eds.). (2002). *Beliefs: A hidden variable in mathematics education?* Dordrecht: Kluwer Academic Publishers.

Lepik, M., & Pipere, A. (2011). Baltic-Nordic Comparative Study on Mathematics Teachers' Beliefs – In: *Teaching Mathematics: Retrospective and Perspectives. 12th International Conference. Abstracts*. iauliai: UL.

Matthews, M. R. (1993). Constructivism and Science Education: Some Epistemological Problems. *Journal of Science Education and Technology*, *2* (1), 359-370.

Morony, S., Kleitman, S., Lee, Y., & Stankov, L. (2013). Predicting achievement: confidence vs self-efficacy, anxiety, and self- concept in Confucian and European countries. *International Journal of Educational Research, 58*, 79-96.

Muijis, D., & Reynolds, D. (2001). *Effective Teaching: Evidence and Practice.* London, Paul Chapman Publishing Ltd, A Sage Publications Company.

OECD. (2004). *Learning for Tomorrow's World: First Results from PISA 2003*. Organisation for Economic Cooperation and Development.

OECD. (2009). *Creating Effective Teaching and Learning Environments: First Results from TALIS*. Paris: OECD Publishing.

Papert, S. (1980). *Mindstorms: Children, Computers and Powerful Ideas.* New York: Basic Books.

Pehkonen, E. (1994). Teachers' and pupils' beliefs in focus – consequence of constructivism. In M. Ahtee, & E. Pehkonen (Eds.), *Constructivist Viewpoints for School Teaching and Learning in Mathematics and Science* (pp. 27-33). Helsinki: University of Helsinki. Department of Teacher Education. Research Report 131.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 257-315). Charlote, NC: National Council of Teachers of Mathematics.
Pipere, A. (2005). Primary and secondary teachers: beliefs and performance related self-perceptions about engaged learning. *Baltic Journal of Psychology*, *6*, 32-44.
Ponte, J. P., Chapman, O. (2006). Mathematics teachers' knowledge and practice. In A. Gutierrez y P. Boero (Eds.). *Handbook of Research of the Psychology of Mathematics Education: Past, Present and Future* (pp. 461-494). Rotterdam: Sense Publishing.
Raymond, A. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching practice. *Journal for Research in Mathematics Education, 28* (5), 550-576
Staub, F. C., & Stern, E. (2002). The nature of teachers' pedagogical content beliefs matters for students' achievement gains: Quasi-experimental evidence from elementary mathematics. *Journal of Educational Psychology*, *94* (2), 344–355.
Suriza van der Sandt. (2007). Research framework on mathematics teacher behaviour: Koehler and Grows' framework revisited. *Eurasia Journal of Mathematics, Science & Technology Education*, *3* (4), 343-350.
Swan, M. (2006). Designing and using research instruments to describe the beliefs and practices of mathematics teachers. *Research in Education*, *75*, 58-70.
Šapkova, A. (2011a). Constructivist beliefs of Latvian mathematics teachers: looking into future. *Journal of Teacher Education for Sustainability*, *13* (1), 99–112.
Šapkova, A. (2011b). Latvian mathematics teachers' beliefs on effective teaching. *International Journal for Mathematics Teaching and Learning*. Retrieved from http://www.cimt.plymouth.ac.uk/journal/shapkova.pdf
Šapkova, A. (2012). Study on Latvian mathematics teachers' espoused beliefs about teaching and learning and reported practice. *International Journal of Science and Mathematics Education*. doi: 10.1007/s10763-012-9361-0.
Šapkova, A. (2013). The dynamic of growth of achievement in mathematics: the analysis of international research. *Acta Paedagogica Vilnensia, 30* (1), 68-79.
Tatto, M. T., Schwille, J., Senk, S. L., Ingvarson, L., Rowley, G., Peck, et all. (2012). *Policy, practice, and readiness to teach primary and secondary mathematics in 17 countries: Findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M)*. Amsterdam: International Association for the Evaluation of Educational Achievement.
Thompson, A. G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics, 1*5, 105-127.
Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grows (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 147-164). New York, NY: Macmillan.

Advised by Laima Railienė, University of Šiauliai, Lithuania

Received: *February 14, 2014*   Accepted: *March 05, 2014*

Aļesja Šapkova	PhD Student, Daugavpils University, Puškina Street 7-19, LV-5400, Daugavpils, Latvia. E-mail: alesja.shapkova@gmail.com