FRACTAL TEXTURE: A SURVEY

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Abstract- The places where you can find fractals include almost every part of the universe, from bacteria cultures to galaxies to your body. Many natural surfaces have a statistical quality of roughness and self-similarity at different scales. Mandelbrot proposed fractal geometry and is the first one to notice its existence in the natural world. Fractals have become popular recently in computer graphics for generating realistic looking textured images. This paper presents a brief introduction to fractals along with its main features and generation techniques. The concentration is on the fractal texture and the measures to express the texture of fractals.

Keywords- fractal dimension, fractal texture, lacunarity, succolarity

Introduction

With the advent of information technology, innumerable digitized images, photos and videos are being produced every day. This exponential growth has created a high demand for efficient tools for image searching, browsing and retrieval from various domains such as remote sensing, fashion, crime prevention, publishing, medicine and architecture.

The analysis of texture is an important topic in the field of image processing and pattern recognition. A variety of statistical methods such as autocorrelation, co-occurrence approach, Law’s method, Markov random field model etc., have been proposed for texture analysis. Fractal-based texture classification is another approach that correlates between texture coarseness and fractal dimension. The limitations and inadequacies of the traditional methods of texture analysis prevent them from fully describing natural textures.

Fractals

A fractal is a rough or fragmented geometric shape that can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole. The term is coined by Benoît Mandelbrot in 1975 and was derived from the Latin word fractus, meaning "broken" or "fractured" [1]. A mathematical fractal is based on an equation that undergoes iteration, a form of feedback based on recursion [2].

Because fractals appear similar at all levels of magnification, fractals are often considered to be infinitely complex (in informal terms). Natural objects that approximate fractals, to a degree, include clouds, mountain ranges, lightning bolts, coastlines, snowflakes, various vegetables (cauliflower and broccoli) and animal coloration patterns. However, not all self-similar objects are fractals, for example, the real line (a straight line) is formally self-similar but fails to have other fractal characteristics; for instance, it is regular enough to be described in Euclidean terms.

A fractal often has the following features:

- It has a fine structure at arbitrarily small scales.
- It is too irregular to be easily described in traditional Euclidean geometric language.
- It is self-similar (at least approximately or stochastically).
- It has a Hausdorff dimension which is greater than its topological dimension.
- It has a simple and recursive definition.

The detailed study of fractals is elaborated by Falconer [3], Rani & Kumar [4,5], Rani and Goel [6] and Singh, et al [7].

Generation of fractals

There are four common techniques for generating fractals

- Escape-Time Fractals

Also known as "orbits" fractals and defined by a formula or recurrence relation at each point in a space. Examples of this type are the Mandelbrot set, Julia set, The Burning Ship fractal, the Nova fractal and the Lyapunov fractal. A Julia set, an example of escape-time fractal is shown in [Fig 1].

- Iterated Function Systems

Denoted by IFS and have a fixed geometric replacement rule. Examples of this type are the Cantor sets, fractal carpet, Sierpinski
gaskets, Peano curve, Koch snowflakes, Harter-Highway dragon curve, T-square, Menger sponge.

- **Random Fractals**
  Generated by stochastic rather than deterministic processes. Examples of these types are trajectories of the Brownian motion, Lacy flight, fractal landscapes and the Brownian tree. In [Fig-2], a fractal landscape is shown.

- **Strange Attractors**
  These kind of fractals are generated by iteration of a map or the solution of a system of initial-value differential equations that exhibit chaos. See a strange attractor in [Fig-3].

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**Fractal Texture**

The fractal dimension gives a measure of the roughness of a surface. Intuitively, the larger the fractal dimension, the rougher the texture is. Pentland [8,9] provided the first theory in this respect by stating that fractal dimension correlates quite well with human perception of smoothness versus roughness of surfaces, with fractal dimension of 2 corresponding to smooth surfaces and fractal dimension 2.9 to a maximum rough “salt-and-pepper” surface [8]. The simplest version of fractal dimension can be described by following relation:

\[ D = \frac{\log N}{\log s} \]

where N is the number of self-similar pieces, s is the linear scaling factor (sizes) of the pieces to the whole and D is the dimension that characterizes the relationship between size and number i.e. the fractal dimension.

Rearranging the elements of the above equation we can solve for D:

\[ D = \frac{\log N}{\log s} \]

The appeal of fractal dimension is not dependent on measurement of scale. It describes the “roughness” of images as natural, the way we perceive roughness [10]. Because fractal dimension alone will not classify all types of natural textures, other features often be used such as lacunarity feature, signatures, directional fractal dimension [11] local fractal dimension [12], extended fractal analysis [13] multifractal analysis [14-16] etc. Since the fractal approach was put forward by Pentland [8,9] other workers have expressed certain problems with it. For example, reducing all textural measurements to the single measure fractal dimension clearly cannot permit all textures to be distinguished [17]. Hence there have been moves to define further fractal-based measures.

**Aspects of Texture**

**Lacunarity**

Mandelbrot himself brought in the concept of lacunarity and in 1982 provided one definition, while Keller, et al [17] and others provided further definitions. When talking about fractals a intuitive definition about Lacunarity is that a fractal is to be called lacunar if its gaps tend to be large, in the sense that they include large intervals (discs, or balls) [1]. If a fractal has large gaps or holes, it has high lacunarity; lacunarity being a measure of spatial heterogeneity can be used to differentiate between images that have similar fractal dimensions but different appearance on the other hand, There are applications of lacunarity in image processing, ecology, medicine and other fields [18,19]. Kermal presents a method to combine fractal dimension and lacunarity for better texture recognition [20]. Tolle gave a qualitative definition about the relationship between fractal dimension and lacunarity [21].

**Succolarity**

Melo’s dissertation [22] is concerned with image processing and analysis. Nevertheless up to now, none has used succolarity to characterize patterns through images. Mandelbrot defines the succolarity of a fractal considering that a succollating fractal is one that “nearly” includes the filaments that would have allowed percolation; since percolare means “to flow through”, succolare (sub-colare) seems the proper term for “to almost flow through” [1]. However, Mandelbrot does not present measures of succolarity in his book. Melo [22] in his thesis extensively studied issues related to estimation of fractal dimension, lacunarity and succolarity and an approach to compute succolarity is also given. The succolarity measures the percolation degree of an image (how much a given fluid can flow through this image).
Other Aspects

Over the years, researchers have studied other features for texture analysis. Many of these features represent the local behavior of the texture via the second-order correlation structure [23], cross sections of the pdf via spatial gray level dependence matrices [24] and exponential random models such as Gaussian Markov random fields (GMRF’s) [25,26]. More recent features represent subbands to provide a compact description of the harmonics in the texture using local linear transformations [27,29] and wavelet packets. Wold features combine harmonic and local statistical features. While the subband techniques succeed in classifying a wide variety of textures, most of the algorithms fail to distinguish between many natural textures that show no periodic structure. These natural textures do not contain any detectable quasiperiodic structure. Instead, they exhibit random but persistent patterns that result in a cloudblike texture appearance. Examples of these “cloudy” textures include mammographic images, terrain models (when the elevation is viewed as a gray level) and pictures of fire, dust, clouds and smoke. To represent these natural textures, Mandelbrot popularized the self-similar fractional Brownian motion (fBm) model, which is characterized by a single parameter known as the Hurst parameter [1]. The Hurst parameter controls the visual roughness of the process at all scales. Because of the ability of the fractal dimension to distinguish texture by roughness [8] researchers have proposed many different fractal texture classification and segmentation algorithms for applications ranging from medicine, remote sensing and target detection, e.g. and references therein [8,9,13,17].

Applications

Texture analysis methods have been utilized in a variety of application domains. In some of the mature domains (such as remote sensing) texture already has played a major role, while in other disciplines (such as surface inspection) new applications of texture are being found. We will briefly review the role of texture in automated inspection, medical image processing, document processing and remote sensing.

In Textile Inspection

There have been a limited number of applications of texture processing to automated inspection problems. These applications include defect detection in images of textiles and automated inspection of carpet wear and automobile paints. In the detection of defects in texture images, most applications have been in the domain of textile inspection. Dewaele, et al [30] used signal processing methods to detect point defects and line defects in texture images.

In Medical Image Analysis

Image analysis techniques have played an important role in several medical applications. In general, the applications involve the automatic extraction of features from the image which is then used for a variety of classification tasks, such as distinguishing normal tissue from abnormal tissue. The textural properties computed are closely related to the application domain to be used.

In Document Processing

One of the useful applications of machine vision and image analysis has been in the area of document image analysis and character recognition. Document processing has applications ranging from postal address recognition to analysis and interpretation of maps. Most image analysis methods proposed to date for document processing are based upon the characteristics of printed documents and try to take advantage of these properties. For example, generally newspaper print is organized in rectangular blocks and this fact is used in a segmentation algorithm.

Remote Sensing

Texture analysis has been extensively used to classify remotely sensed images. Land use classification where homogeneous regions with different types of terrains (such as wheat, bodies of water, urban regions, etc.) need to be identified is an important application. Haralick, et al [24] used gray level co-occurrence features to analyze remotely sensed images. Rignot and Kwok [32] have analyzed SAR images using texture features computed from gray level co-occurrence matrices. Du used texture features derived from Gabor filters to segment SAR images [31]. He successfully segmented the SAR images into categories of water, new forming ice, older ice and multi-year ice. Lee and Philpot also used spectral texture features to segment SAR images [16].

Conclusion

In this survey paper we have attempted to review the concept of fractals and fractal texture. Concept of fractals is explained along with its features and generation techniques. Main focus was on the texture which is a surface that exhibits local roughness or structure, which is then projected to form a textured image. Different measures such as lacunarity and succolarity were also discussed to express the texture of fractals. We have also discussed the role and applications of texture in different fields.

References

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