An Efficient Hybrid MoM/FEM Method for Analyzing the Enclosures With Apertures

Açıklıklı Kutuların Analizinde Etkin Bir Hibrid MoM/FEM Yöntemi

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1. INTRODUCTION

With growing functionality of modern communication systems and enhancements in mobility and automatization, the relevant equipments become more compact and more complex. To prevent from harms or to provide electromagnetic protection, electronic systems should be placed into conducting enclosures. These conducting enclosures are mostly referred as shielding enclosures. On the walls of these shielding enclosures, there may exist some apertures which can cause a significant coupling between the circuitry in the enclosure and the outer environment. So there would be interference between the fields entering from the apertures and circuitry inside the enclosure, therefore shielding efficiency is affected significantly by those apertures.

The most important phase in designing a shielding enclosure is to minimize the effects of such apertures on shielding efficiency. To know shielding efficiency, one must predict EM fields inside the enclosure. Shielding effectiveness (SE) is an important parameter which reflects the shielding efficiency of the devices. It is defined in terms of the ratio of the observed fields in the absence of the shield, to the observed field in the presence of the shield measured at the same point and it is expressed in dB (Siah et al., 2003). For evaluating the interference, all electromagnetic fields should be calculated. Various analytical and numerical techniques have been developed to determine EM fields inside the shielding enclosure. The analytical formulation which calculates the shielding effectiveness of an empty enclosure is presented

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by (Robinson et al., 1998). However, this formulation is valid only for rectangular boxes and is limited by the assumption of single mode. For complex enclosure configuration, to find EM fields analytically is very difficult. Thus the use of numerical methods is indispensable.

The electromagnetic radiation from slots and apertures in shielding enclosures is studied experimentally and using FDTD technique by (Li et al., 2000). In (Olyasger et al., 1999), the shielding effectiveness of enclosures is performed experimentally and numerically with an electromagnetic simulator based on method of moments. (Rajamani and Bunting, 2006) are presented a Modal/MoM method, which calculates the shielding effectiveness of an empty rectangular enclosure with rectangular apertures. In (Wang et al., 2002), the EM coupling of plane wave penetrating through apertures is examined with FDTD method and the electric field distribution inside the enclosure is obtained. A study of an enclosure with aperture using finite element time domain with a mass lumping technique was investigated by (Benhassine et al., 2002). The problem of calculating SE of a rectangular enclosure with perforated walls was formulated by (Deshpande, 2000) by replacing the apertures with equivalent magnetic current sources and representing the fields radiated by such sources in terms of cavity Green’s functions. In (Carpes et al., 2002), the coupling of plane wave with a conducting wire placed inside a metallic cavity is examined using frequency and time domains FEM and induced voltage on wire is computed. In (Feng and Shen, 2005), a hybrid technique combining the finite difference method and the MoM is proposed to compute the shielding effectiveness of rectangular enclosure with apertures. To evaluate the shielding efficiency of metallic rectangular enclosures, (Wallyn et al., 2002) proposed a new MoM technique solving a mixed potential integral equation. A hybrid technique combining the finite element method and the MoM is presented to solved electromagnetic radiation problems from structures consisting of an inhomogeneous dielectric and perfectly conducting materials (Ali et al., 1997).

In this paper, a hybrid method is proposed in computing the field leaking through the aperture into a resonator with a rectangular cross-section. In this numerical model, MoM and FEM are combined. The interior and exterior regions of the enclosure are analyzed separately by employing the field equivalence principle. Internal electromagnetic fields are discretized using the finite element method while external fields are formulated by MoM. The hybrid technique takes the advantage of finite element method’s versatility and the method of moment’s high efficiency. In this way the finite element method is applied only inside the metallic enclosure and no absorbing boundary conditions (ABCs) are needed. Use of the FEM to evaluate the internal fields allows us to treat complex objects inside the enclosure, as printed circuit board (PCB), dielectric elements, etc.

2. FORMULATION OF THE PROBLEM

Assume the surface of the enclosure with the aperture as an infinite-width perfect-conductor ground plane, so, this problem could be separated into two regions by Schelkunoff equivalence principle (Rao et al., 1982). The first region represents the inner volume and the second represents the free half-space limited by ground plane.

2.1. Finite Element Method (FEM)

The finite element formulation for the inner region is initialized by applying the Galerkin Procedure to the vector wave equation which depends on the frequency defining the electric field. The problem domain is discretized with tetrahedral elements. Electric field, in the discretized domain, can be expressed as:

$$\vec{E} = \sum_{n=1}^{N} \vec{w}_n \epsilon_n$$  \hspace{1cm} (1)

where $\epsilon_n$ is unknown coefficient associated with edge of the element, $\vec{w}_n$ is basis function and N is degree of freedom. After discretisation, the wave equation changes into the following matrix equation:

$$[S] + j \omega \mu_0 [T_1] - \omega^2 \mu_0 [T_2] \epsilon = [b]$$  \hspace{1cm} (2)

Where $[S]$, $[T_1]$, $[T_2]$ are finite element matrices. Clearly, the expression of the element matrices can be written as following:

$$[S] = \int_{V_i} \nabla \times \vec{\omega}_i \cdot \nabla \times \vec{\omega}_i \, dV$$  \hspace{1cm} (3a)

$$[T_1] = \int_{V_i} \sigma \vec{\omega}_i \cdot \vec{\omega}_i \, dV$$  \hspace{1cm} (3b)

$$[T_2] = \int_{V_i} \epsilon_0 \epsilon_0 \vec{\omega}_i \cdot \vec{\omega}_i \, dV$$  \hspace{1cm} (3c)

$$[b] = \int_{S} j \omega \mu_0 \int_{V_i} \vec{H} \, dS$$  \hspace{1cm} (3d)
Here, \( V_e \) is the integral over a tetrahedral element and contains the boundary condition on the aperture. As the equivalence field theorem, an aperture placed on a perfect-conductor plane is equivalent to a magnetic current distribution. The EM radiation from the aperture to either the free-space or to the inside of the enclosure is equivalent to the radiation which is caused by that magnetic current source.

### 2.2. Moment Method (MoM)

The tangential magnetic field on the aperture can be determined by applying the boundary conditions. The tangential magnetic field on the aperture should be continuous. Therefore, using the boundary condition which the tangential components are equal to each other, we can have the following equation.

\[
\hat{n} \times \vec{H}_{\text{inc}} + \hat{n} \times \vec{H}_{\text{ext}} = \hat{n} \times \vec{H}_{\text{tot}}
\]  
\( (4) \)

The unknown tangential magnetic field on the aperture boundary surface \( \hat{n} \times \vec{H} \) would be

\[
\hat{n} \times \vec{H} = \sum_{n} n \, \vec{f}_n
\]  
\( (5) \)

and this expression can be placed into \( (3d) \) equation. Here, \( \vec{f}_n \), \( n \) the basis function and \( J_n \) is the amplitude of this basis function. For this boundary surface, a relation exists which can be expressed by (Jin, 1993).

\[
\vec{w}_n = \hat{n} \times \vec{f}_n
\]  
\( (6) \)

The integral equation, which is an inner product of the equality in \( (3) \) and the test function selected using Galerkin method, could be transformed into the following matrix form:

\[
\begin{bmatrix} \vec{h}_{\text{inc}} \end{bmatrix} + \begin{bmatrix} \vec{Y}_{\text{cout}} \end{bmatrix} \begin{bmatrix} \epsilon \end{bmatrix} = \begin{bmatrix} \vec{Y}_{\text{inout}} \end{bmatrix} \begin{bmatrix} \vec{J}_n \end{bmatrix}
\]  
\( (7) \)

Here, \( \{ \epsilon \} \) is the unknown electric field amplitude vector on the aperture, \( \begin{bmatrix} \vec{h}_{\text{inc}} \end{bmatrix} \), \( \begin{bmatrix} \vec{Y}_{\text{cout}} \end{bmatrix} \) and \( \begin{bmatrix} \vec{Y}_{\text{inout}} \end{bmatrix} \) are the matrix which could be achieved by using the inner products of magnetic field on the aperture. Consequently, the integral equation which depends on the unknown electric field on the aperture is turned into a matrix equation using MoM.

Next step is to place the right-hand side of the matrix equation into the system equation of the finite element method. If \( (5) \) is placed into the right hand-side of \( (3d) \), \( \{ b \} \) would be as following

\[
b = j \omega \mu \sum_{j} \int_{S} \vec{w}_j \cdot \vec{f}_j \, dS
\]  
\( (8) \)

If we rewrite \( \{ b \} \) as \( \{ b \} = [B] \cdot \{ J \} \) then the frequency domain finite element matrix of \( (2) \) would be as following:

\[
([S] + j \omega \mu_0 [T] \cdot \omega \mu_0 [J]) \{ \epsilon \} = [B] \{ J \}
\]  
\( (9) \)

### 3. FINDINGS

In this section, we present some numerical results obtained using the hybrid MoM/FEM technique described in the previous sections. To validate efficiency of the numerical model, the hybrid method is applied to empty enclosure with aperture. The dimensions of the rectangular enclosure is chosen as \( A=30 \text{ cm}, B=12 \text{ cm} \) and \( C=30 \text{ cm} \). The aperture whose length and width are 10 cm and 0.5 cm is located at the center of front wall \( (x_0=15 \text{ cm} \text{ and } y_0=6 \text{ cm}) \). It is considered that the enclosure is illuminated by a y-polarized plane wave impinging normally on the aperture in the front wall. The geometry of empty rectangular enclosure with aperture is shown in Figure 1.

In FEM mesh, resonator is separated into 7x3x6 hexas, and each hexa is separated into 5 tetras. The unknown number in the mesh is 1015. With the presented MoM/FEM hybrid method, the electric field distribution into the resonator is calculated. The method is applied to rectangular enclosure with rectangular aperture in the front wall. The walls of the enclosure are assumed to be thin and perfect conductor. While the SE produces point results for the design and optimization aspects, the stored electrical energy (SEE) which represent the electric field distribution inside the whole resonator must be take into account. Relative stored electrical energy is given by (Siah et al., 2003).
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Figure 1. Geometry of rectangular enclosure with aperture.

\[ \text{Stored electrical energy ratio (dB) = } 10 \log \left( \frac{\iiint e_\text{total} F^2 \, dV}{\iiint e_\text{inc} F^2 \, dV} \right) \tag{10} \]

Where, \( \langle E_{\text{total}} \rangle \) refer to the total fields computed at the calculation point inside of the enclosure. \( \langle E_{\text{inc}} \rangle \) refers to the incident field computed at the same location in the absence of the enclosure.

Figure 2. Simulations results for the shielding effectiveness at the center of enclosure with aperture (10x0.5cm).

The shielding effectiveness at center of the enclosure is calculated with the method and the results is compared by (Robinson et al., 1998) (Figure 2). As shown in Figure 2, the results are of very good agreement.

Figure 3. The change of the electrical energy stored in an empty resonator with aperture with frequency (l=20cm and w=1cm).

The ratio of stored electrical energy (Figure 3) with frequency inside empty enclosure is examined. The dimension of the aperture is 20x1cm. In figure 3, the change of the electrical energy stored in an empty resonator with aperture with frequency is shown. The aperture with 20x1 cm dimensions is located on the front surface of the resonator. The results we obtained is compared with the results of (Siah et al., 2003) and a good agreement is achieved.

Figure 4. The affect of the change of the aperture width on the stored electrical energy.

In Figure 4, stored electrical energy results obtained from (11) under different aperture widths are shown. It can be seen that less electrical energy is stored in the resonator volume with narrower apertures. Wider aperture decreases the shielding performance, as well as increase the electrical energy stored in the resonator.
4. CONCLUSION

In this study, the SEE and SE of enclosure with aperture has been investigated using hybrid MoM/FEM in frequency domain. The results for enclosure in the literature matches with the results achieved with MoM/FEM. The SEE with different aperture sizes is studied. By adjusting the aperture size, one can effectively control the low-frequency SE and SEE characteristics. The method can be applied to a variety of problems that involves the coupling between metallic enclosures through aperture. The use of the FEM allows the potential application of the hybrid method to very complex geometry, in a very efficient way and without the absorbing boundary condition. This is due to the application of the equivalence principle over the aperture of the enclosure. Numerical results have shown the validation of the hybrid technique in modeling the shielding effectiveness of empty enclosure. By changing aperture size, the optimum configuration for EMC, which stores minimum energy, can be found. The method can be readily extended to evaluate the SE and SEE of enclosure with other geometries.

5. NOMENCLATURE

\( \vec{E} \) : Electric field,
\( \sigma \) : Conductivity,
\( \vec{v}_i \) : Test function,
\( c_i \) : Unknown coefficient associated with i. edge of the element,
\( \vec{v}_j \) : Basis function associated with i. edge of the element,
\( N \) : Degree of freedom,
\( \alpha_i \) : Unknown coefficient associated with i. edge in aperture,
\( \vec{J}_i \) : Basis function associated with i. edge in aperture,
\( N_i \) : Total edge numbers in aperture,
\( [S] \) : Finite element matrices,
\( \Gamma \) : Finite element matrices,
\( [B] \) : Finite element matrices,
\( \mu_0 \) : Permeability of free space,
\( \varepsilon_0 \) : Permittivity of free space,
\( J_m \) : Magnetic current distribution,
\( H^{\text{ext}} \) : The magnetic field of the radiation to the outer environment by \( J_m \),
\( H^{\text{int}} \) : The magnetic field on the aperture that is radiated into enclosure by \( J_m \),
\( [V^{\text{inc}}] \) and \( [V^{\text{ext}}] \) : The matrices representing the inner products of magnetic fields on the aperture,
\( V_e \) : Integral over a tetrahedral element,
\( S_a \) : Aperture surface.
REFERENCES


