SOME NEW DİFFERENCE SEQUENCE SPACES DEFİNED BY A SEQUENCE OF MODULI

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Özet- **Bu çalışmada, modülüs fonksiyon dizisi yardımıyla bazı yeni fark dizi uzayları tanımlanarak bunların birtakım özellikleri çalışıldı.**

Anahtar kelimeler- **Dizi uzayları, fark dizileri, modulus fonksiyonu.**

Abstract- **The main object of this paper is to introduce and study some difference sequence spaces defined by using a sequence of moduli .**

Key words- **Sequence spaces, difference sequences, modulus functions.**

I.Introduction

Let w be the space of all real (or complex) $x=(x_k)$ sequences and , c_0 and c denote respectively the Banach spaces of bounded,null and convergent sequences, normed as usual by $\|x\| = \sup_k |x_k|$. A sequence x∈ is said to be almost convergent [6] if all Banach limits of x coincide. Lorentz[6] proved that x is almost convergent to s if and only if

$$\lim_{m\to\infty} m^{-1} \sum_{i=n+1}^{n+m} x_i = s \text{ , uniformly in n .}$$

Let denote the space of all almost convergent sequences.
Maddox[2][3] has defined x to be strongly almost convergent to number s if

$$\lim_{m\to\infty} m^{-1} \sum_{i=n+1}^{n+m} | x_i - s| = 0 \text{ , uniformly in n .}$$

By [ ], we denote the space of all strongly almost convergent sequences.it is easy to see that [ ] ⊂ ⊂ .

Several authors have discussed the spaces of strongly almost convergent sequences. The class of sequences which are strongly almost convergent with respect to a modulus was introduced by Pehlivan[8] as an extension of the definition of strongly almost convergence. Esi [1] extended strongly almost convergent sequence spaces to w [A,p,F], w[A,p,F] and w_0[A,p,F] for $p=(p_k)$ with $p_k>0$, nonnegative $A=(a_{nk})$ regular matrix and a sequence of moduli , which generalized the spaces [F (f)], [F(f)] and [F_0(f)] of Pehlivan [8].

We recall that a modulus function f is a function from [0,∞) to [0,∞) such that

(i) f(x)=0 iff x=0,
(ii) f(x+y) ≤ f(x)+f(y) for all x,y≥0,
(iii) f is increasing ,
(iv) f is continuous from the right at zero.

A modulus may be bounded or unbounded (Ruckle[9] and Maddox[5]).

In this paper we introduce and examine some new difference sequence spaces by using a sequence of moduli. Let F be a sequence of moduli and $p=(p_k)$ be a sequence of strictly positive real numbers and suppose that $A=(a_{nk})$ be a nonnegative regular matrix. We define

w[A,p,F]={ x∈w :

$$\lim_{m\to\infty} \sum_{k=1}^{\infty} a_{mk}[f_k | \ x_{k+n} - s |]^{p_k} = 0 \text{ , uniformly in n,}$$

for some s }

w_0[A,p,F]={ x∈w :

$$\lim_{m\to\infty} \sum_{k=1}^{\infty} a_{mk}[f_k | \ x_{k+n} |]^{p_k} = 0 \text{ , uniformly in n } \}$$

w [A,p,F] = { x∈w :

$$\sup_{m,n} \sum_{k=1}^{\infty} a_{mk}[f_k | \ x_{k+n} |]^{p_k} = 0 \text{ , uniformly in n } \}$$

where x = (x_k) = ($x_k - x_{k+1}$) and for convenience, we put f(| x_k|) ^{p_k} instead of {f(| x_k |)} ^{p_k}.
Let E be any of the spaces w[A,p,F], w_0[A,p,F] and w [A,p,F] . Then it is easy to see that E ⊂ E.

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w [A,p,F] . Then it is easy to see that E ⊂ E.

When f_k=f and p_k=1 for all k, we denote these sequence spaces by w[A,p], w_0[A,p] and w [A,p]. If x∈ w[A,p] we say that x=(x_k) is -strongly almost A-summable to s with respect to the modulus f. If p_k=1 for all k , we write w [A,F], w[A,F] and w_0[A,F] for w [A,p,F], w[A,p,F] and w_0[A,p,F], respectively.

When A=(a_{mk})=(C,1) Cesaro matrix and f_k=f for all k, we obtain following sequence spaces.

[F(f,p)]={ x∈w :

$$\lim_{m\to\infty} m^{-1} \sum_{i=n+1}^{n+m} [f | x_i - s |]^{p_i} = 0 \text{, uniformly in n,}$$

for some s }

[F_0 (f,p)]={ x∈w :

$$\lim_{m\to\infty} m^{-1} \sum_{i=n+1}^{n+m} [f | x_i |]^{p_i} = 0 \text{, uniformly in n }\}$$

$$[F \ (f,p)]=\{x\in w : \sup_{m,n} m^{-1} \sum_{i=n+1}^{n+m} [f | x_i |]^{p_i} < \ \}$$

Note that if A=(C,1) Cesaro matrix, p_k=1 and $f_k(x)$ = x for all k, then w[A,p,F] = ( ) = { x : x∈ } . Also in this case w [A,p,F] = ( ) = { x : x∈ }.

For a sequence of moduli F=(f_k) , we give following conditons;

(1) $\sup_k f_k(t) <$ for all t > 0 ,

(2) $\lim_{t\to\infty} f_k(t) = 0$ uniformly in k≥1.

We remark that in case f_k=f for all k, where f is a modulus,the conditons (1) and (2) are automatically fulfilled.

II.Main Results.

Theorem 1. Let p=(p_k) be bounded. Then w [A,p,F] , w[A,p,F] and w_0[A,p,F] are linear spaces of the complex field.

Proof. Let $\sup_k p_k$= H. If a_k,b_k and α are complex numbers, then we have [2,p.346]

(3) $| a_k+b_k |^{p_k}$ max $(1,2^{H-1})$ $(| a_k |^{p_k} + | b_k |^{p_k})$

and

(4) $| \alpha |^{p_k}$ max $(1, | \alpha |^{H})$

The result follows from (3) and (4).

Theorem 2. Let A be a nonnegative regular matrix and F=(f_k) be a sequence of moduli. If (1) holds then

w[A,p,F] ⊂ w [A,p,F]

Proof. It is a direct consequence of (3).

Theorem 3. w_0[A,p,F] and w[A,p,F] are linear topological space paranormed by g defined by

$$g(x)= \sup_{m,n} \{ \sum_{k=1}^{\infty} a_{mk}[f_k | x_{k+n} |]^{p_k} \}^{1/M}$$

where H=sup p_k<∞, M=max(1,H) .

Proof. From Theorem 2 , for each x∈ w[A,p,F], g(x) exists. Clearly g(0)=0, g(x)=g(-x). Take any x,y∈ w[A,p,F] . Since p_k / M 1 and M≥1, using the Minkowski's inequality and definition of f , we have g(x+y) g(x)+g(y). To check the continuity of multiplication, let us take any complex λ and x∈ w[A,p,F]. Whence λ→0, x→0 imply g(λx) →0 and also x→0 , λ fixed imply g(λx) →0 . We now show that λ→0 , x fixed imply g(λx) →0 . As m→∞, let

$$b_{mn}= \sum_{k=1}^{\infty} a_{mk}[f_k (| x_{k+n} - s |)]^{p_k} \to 0\text{,uniformly in n.}$$

For |λ|<1 we have

$$\{ \sum_{k=1}^{\infty} a_{mk}[f_k (| \lambda \ x_{k+n} |)]^{p_k} \}^{1/M}$$

$$\{ \sum_{k>n} a_{mk}[f_k (| x_{k+n} - s |)]^{p_k} \}^{1/M} +$$

$$\{ \sum_{k\le N} a_{mk}[f_k (| \lambda \ x_{k+n} - \lambda s |)]^{p_k} \}^{1/M} +$$

$$\{ \sum_{k=1}^{\infty} a_{mk}[f_k (| \lambda s |)]^{p_k} \}^{1/M}$$

Let ε>0 and choose N such that for each n,m and k>N implies b_{mn} < ε/2. For each N, by continuity of f_k for all k, as λ→0,

$$\{ \sum_{k\le N} a_{mk}[f_k (| \lambda \ x_{k+n} - \lambda s |)]^{p_k} \}^{1/M} +$$

$$\{ \sum_{k=1}^{\infty} a_{mk}[f_k (| \lambda s |)]^{p_k} \}^{1/M} \to 0$$

Then choose δ<1 such that |λ|< δ implies

$$\{ \sum_{k\le N} a_{mk}[f_k (| \lambda \ x_{k+n} - \lambda s |)]^{p_k} \}^{1/M} +$$

$$\{ \sum_{k=1}^{\infty} a_{mk}[f_k (| \lambda s |)]^{p_k} \}^{1/M} < \varepsilon/2$$

Hence we have

$$\{ \sum_{k=1}^{\infty} a_{mk}[f_k (| \lambda \ x_{k+n} |)]^{p_k} \}^{1/M} < \varepsilon$$

g(λx) →0 (λ→0). Thus w[A,p,F] is paranormed linear topological space by g.

Theorem 4. Suppose that A be a nonnegative regular matrix and $F=(f_k)$ be a sequence of moduli then

(i) ( ) $\subset$ w [A,p,F].

(ii) If $0<p_k \leq q_k$ for all k and (q_k/p_k) bounded, $w[A,q,F] \subset w[A,p,F]$.

(iii) If (1) and (2) hold then $w[A,p] \subset w[A,p,F]$.

(iv) If $\beta = \lim_{t\to\infty} (f_k(t)/t) > 0$ for all k then $w[A,p] = w[A,p,F]$.

Proof. (i) is trivial.

(ii) If we take $w_{k,n} = [f_k(|x_{k+n} - s|)]^{p_k}$ for all k and n then using the same technique of Theorem 2 of Nanda [7] it is easy to prove (ii).

(iii) Using the same technique of Theorem 4 of Maddox [4] it is easy to prove this.

(iv) We must show $w[A,p,F] \subset w[A,p]$. For any modulus function, the existence of positive limit given with β was given in Maddox[5]. Now $\beta>0$ and let $x \in w[A,p,F]$. Since $\beta>0$, for every $t>0$ we write $f_k(t) \geq \beta t$ for all k. From this inequality , it is easy to see that $x \in w[A,p]$.This completes the proof.

Theorem 5. Suppose that $F=(f_k)$ and $G=(g_k)$ be a sequences of moduli and $g_k \geq f_k$ for all k then $\lim_{x\to\infty} [f_k(x)/g_k(x)] <$ implies $w[A,p,G] \subset w[A,p,F]$.

Proof. It is trivial.

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